Discrete Math Final Exam, Take-home part

Math/CS 150, Jason Grout, Spring 2013

Write up complete answers to the exam problems. Your answers should explain the reasoning behind your computations. Turn in this signed sheet, as well as your answers to the problems, to my office in Howard 203A (slip it under the door if I'm not there). This part of the exam is due by 4:30pm on Friday, 17 May.

Your exam may be turned in once before your final submission to get feedback on your answers (like whether you are being comprehensive and clear enough, if you are missing something big, etc.). I will give you that feedback within 24 hours of me receiving your draft.

While completing this exam, you may refer to

1. The class textbook
2. Your own class notes and your own solutions to problems
3. Your calculator

You *may not* use *any* other materials/resources. The following list is not exhaustive, but is a reminder of some specific things you're not allowed to use in working on this exam:

1. You *may not* use the Internet
2. You *may not* use the work (including class notes) of other students (whether those students are in the class this semester or not).
3. You *may not* use books other than the official class textbook.

After completing the exam, sign the following statement to indicate that you have followed these instructions:

> I certify that all of the answers on this exam are the result of my own efforts. I have not consulted with others nor used any sources not specifically allowed by the instructions on this page in the completion of any part of this exam. I have not provided help to any others for this exam.

Name (printed): ______________________________

Signature: ______________________________ **Date:** ____________

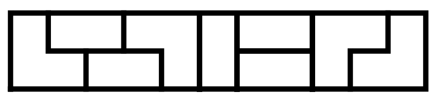
1. A sequence of A, G, C, T letters represents a DNA strand.

    (a) How many different DNA strands have three A, two G, four C, and five T letters?

    (b) How many different DNA strands have three A, two G, four C, and five T letters and contain the sequence AA?

2. Let S be a set of more than half of the numbers in the range $1, 2, \ldots, 2n$. Show that no matter what the integer n is or how S is chosen, one of the numbers in S is a multiple of another number in S.

3. In class we considered the number of tilings of a $2 \times n$ strip using 2×1 domino pieces, and showed that this is counted by the Fibonacci numbers. We now consider the problem of determining the tilings of a $2 \times n$ strip using 2×1 domino pieces and "L" pieces that are composed of 3 square units. Let a_n be the number of ways to tile a $2 \times n$ strip using these pieces. An example of a tiling of the 2×11 strip is shown below. By convention we will let $a_0 = 1$. (Thanks to Steve Butler for this problem.)



    (a) Determine a_1, a_2, a_3 and a_4 by drawing all possible tilings.

    (b) Show for $n \geq 3$
    $$a_n = a_{n-1} + a_{n-2} + 2a_{n-3} + 2a_{n-4} + \cdots + 2a_0.$$
    (Hint: what can the set of pieces forming the last "block" look like.)

    (c) Use the relationship from the previous part to show for $n \geq 3$ that
    $$a_n = 2a_{n-1} + a_{n-3}.$$