Which of the following sets are vector spaces (i.e., spans)? Do the following for each set below:

- Write 3 elements of the set (if possible)
- If the set is a vector space, write the set as a span of some vectors in the space (i.e., “parametrize” the set).
- If the set is not a vector space, give a specific example showing that the set is not closed under vector addition or not closed under scalar multiplication.

1. “vector” is a matrix

   (a) All matrices with 3 rows and 2 columns (i.e., all $3 \times 2$ matrices)
   (b) All $3 \times 3$ matrices where every entry not on the diagonal is zero
   (c) All $3 \times 3$ matrices where every entry below the diagonal is zero
   (d) All $3 \times 3$ matrices that are symmetric (ask me what this means)
   (e) All $3 \times 3$ matrices that have an RREF of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
   (f) All $2 \times 2$ matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a + d = 0$
   (g) All $2 \times 2$ matrices $\begin{bmatrix} a & b \\ c & 0 \end{bmatrix}$ where $a + b + c = 0$
   (h) The single $2 \times 2$ zero matrix

2. “vector” is a polynomial

   (a) All polynomials with degree at most 2
   (b) All polynomials with degree exactly 2
   (c) All polynomials with degree at most 3 and having integer coefficients
   (d) All polynomials with the form $\{a + ct^2 \mid a, c \in \mathbb{R}\}$
   (e) All polynomials of the form $a + bt + ct^2$ where $a + c = 5$.
   (f) All polynomials with degree at least 2
   (g) All polynomials (with any degree)

3. “vector” is a real function (takes in one number, gives back one number)

   (a) All real functions where $f(0) = 0$
   (b) All real differentiable functions where $f'(0) = 1$
   (c) The single function $f(x) = 0$