Linear Algebra Exam 2, Math 80, Jason Grout, Spring 2012

Name: ____________________

> Answer the questions in the spaces provided on the question sheets. Circle all your final answers. Be clear and complete in your reasoning and answers; when in doubt, write it out.

Justify all answers

1. Suppose A and B are 2 by 2 matrices, and $\det(A) = 3$ and $\det(B) = 5$. What is $\det(3A^T B^{-1})$? Show all work justifying the answer.

2. Suppose A is a 5 by 5 matrix which is not invertible. Explain why A must have an eigenvalue of 0.

3. Let V be the set of all polynomials in P_2 such that $p(2) = 0$ (i.e., all polynomials that give 0 when you plug in $x = 2$). Is V a vector space? Explicitly check the conditions for a vector space.

4. Let $V = \{(x, y) \text{ where } x, y \in \mathbb{R}\}$. Define addition on V by $(a, b) \oplus (c, d) = (a + c - 2, bd)$. (Note that I'm writing vector addition as $\oplus$ to make it clear when we are using this new definition). Does V have a zero vector (a vector $\vec{0}$ so that $\vec{v} \oplus \vec{0} = \vec{v}$ for every $\vec{v} \in V$)? If so, what is it? If not, why not?

CALCULATOR PORTION   Name:__________________________

Show *all* work, including inputs and outputs on the calculator. Indicate which computations were done on a calculator.

5. Let $D\colon P_4 \to P_4$ given by $D(p) = p''$, the second derivative of p.

   (a) Is D a linear transformation? Explicitly check that D has the right properties to be a linear transformation.

   (b) What is the kernel of D? Is D injective (one-to-one)? Justify and explain your answers.

   (c) What is the image of D? Is D surjective (onto)? Justify and explain your answers.

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(d) Let $\{1, x, x^2, x^3, x^4\}$ be a basis for P_4. What is the matrix for T relative to this basis (i.e., a matrix A so that if $\vec{x}$ is a coordinate vector for a polynomial p, then $A\vec{x}$ is the coordinate vector for p'')? Check your work by using your matrix to calculate the second derivative of $3x^4+2x^3-2x^2+x-1$.

6. In this problem, a "vector" is a 2 by 2 matrix. Again, explicitly show the inputs and outputs of any calculator work you do. Let

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

(a) Is the set $\{A, B, C, D\}$ linearly independent? Why or why not?

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(b) Give a basis for the vector space $V = \text{span}\{A, B, C, D\}$. Call this basis $\mathcal{B}$. [Hint: Since "vectors" in V are 2 by 2 matrices, $\mathcal{B}$ should be a set of 2 by 2 matrices.]

(c) What is the dimension of the vector subspace $V = \text{span}\{A, B, C, D\}$?

(d) Write the coordinates $[A]_{\mathcal{B}}$, $[B]_{\mathcal{B}}$, $[C]_{\mathcal{B}}$, and $[D]_{\mathcal{B}}$ (i.e., the coordinates for each matrix relative to your basis $\mathcal{B}$ from above).