

Linear Algebra Exam 1

Math 80, Jason Grout, Spring 2012

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page and clearly indicate that you have done so. Circle all your final answers. Be clear and complete in your reasoning and answers; when in doubt, write it out.

Justify all answers. No calculator until last problem.

Name: _____

1. Let A , B , C , and D be invertible 5 by 5 matrices such that $ABC^{-1} = D$. Solve for B . Show each individual step of your computation.

2. Suppose that the *augmented* matrix for a system $A\vec{x} = \vec{b}$ of linear equations has been reduced by row operations to the reduced row-echelon form below. Write the form of the solution as a linear combination of vectors (possibly with variables as some of the coefficients).

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right].$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} =$$

3. Find the eigenvalues of $A = \begin{bmatrix} 2 & 4 \\ -3 & -6 \end{bmatrix}$.

4. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$.

(a) Find A^{-1} using row reduction or show that the inverse does not exist.

(b) Check your work by multiplying AA^{-1} . Explain how it serves as a check of your work.

5. The row reduced echelon form of

$$A = \begin{bmatrix} -1 & 3 & -5 & 4 & 18 \\ 1 & -2 & 4 & 0 & -7 \\ 2 & 0 & 4 & -3 & -8 \\ 5 & 1 & 9 & 2 & 2 \end{bmatrix} \quad \text{is} \quad \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis for the row space of A .

(b) Find a basis for the column space of A .

(c) Write each non-pivot column of A as a linear combination of the pivot columns of A (note that these are all columns in the original matrix A).

(d) Find a basis for the null space of A .

(e) Find the rank of A .

Calculator Portion: You may use your calculator on this problem. Please give the rest of the test back to the instructor before pulling out your calculator.

6. Find a parabola that passes through the points $(1, 2)$, $(2, 2)$, $(3, 4)$. To do this problem, set up a system of equations, solve the system using row reduction, and interpret your answer to find the equation of the parabola.