Answer each question. Show all work. In a few places, I suggest you use a computer to help you in your computations. If you do, then show what you put into the computer and what you received out (for example, if you use a computer to calculate an rref, write down the matrix you put into the computer and the resulting rref, and mention that the computer calculated the rref).

1. Consider the vector field \( \vec{F}(x, y) = (x + y, 2x) \). Construct a rough sketch of the vector field (plotting at least 4 vectors). Add to your sketch the directions represented by the eigenvectors and include a few flow lines to illustrate the path of motion.

2. Find the constants \( a_0 \) and \( a_1 \) of the least squares regression line \( y = a_0 + a_1 x \) for the points \((-1, 1), (2, -2), \) and \((0, 2)\). You may use a computer to do the rref, if you would like.
3. Find the critical points of the function $x^3 - 3x^2 + y^2 + 2y + 1$, and use the second derivative test (involving matrix derivatives) to determine if each critical point is a maximum, minimum, or saddle point.

4. Consider the following Markov process. In a factory, the proportion of working parts which break down each month is 5%. The proportion of broken parts which get fixed each month is 90%. Assume these percentages remain constant each month. Currently, 98% of the parts in the factory are working properly. If this process continues each month, eventually what percentage of the parts in the factory will be broken each month (i.e., what is the percentage in the long run)? (again, you might use a computer to help you in your computations)
5. Consider
\[ A = \begin{bmatrix} 1 & 3 & 0 & -4 \\ 2 & 6 & 2 & -4 \\ -1 & -3 & -3 & -2 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 3 & 0 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad A^T \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \].

(a) Give two different bases for the column space of \( A \). Next to each basis, state the coordinates of \((-4,-4,-2)\) relative to your basis.

(b) Give a basis for the null space of \( A \). What is the dimension of the null space?