Name: ________________________________

1. What is the angle between $(1, -3)$ and $(9, 3)$?

2. Give a unit vector that points in the same direction as $(-1, 3, 3)$.

3. Let $B = \{(1, -1), (4, -5)\}$ be a basis for $\mathbb{R}^2$, and let $\vec{x} = (2, 1)$. Find the coordinate vector $[\vec{x}]_B$, the coordinate vector of $\vec{x}$ relative to $B$. 
4. Let $A$, $B$, $C$, and $D$ be invertible $n \times n$ matrices for which $A = CD^{-1}B^{-1}$. Solve this equation for $B$.

5. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given reduced row-echelon form. Solve the system (give all solutions) if it is consistent or tell what is wrong if the system is not consistent.

\[
\begin{bmatrix}
1 & 2 & 0 & 2 & 0 & 2 \\
0 & 0 & 1 & -1 & 0 & 3 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

6. Is the set \{ $x^2 + 2x + 1$, $-x^2 - 2$, $2x^2 + x - x$, $-x^2 + 3x + 1$ \} a linearly independent set in the vector space $P_2(x)$? Justify your answer.
7. Set up a matrix equation $A\vec{u} = \vec{b}$ whose solution $\vec{u}$ gives the coefficients $u_0, u_1, u_2$ for the parabola $y = u_0 + u_1x + u_2x^2$ that passes through the points $(1, 2), (2, -1), (3, 3)$. You do not need to solve this system.

8. Let $A$ be an $n \times n$ matrix. Let $S$ be the set of all vectors $\vec{v}$ in $\mathbb{R}^n$ such that $A\vec{v} = -\vec{v}$. Either show that $S$ is a vector subspace of $\mathbb{R}^n$ or give a reason why $S$ is not a subspace. Justify your answer completely.
9. For this page and the next page, consider the following matrix

\[
M = \begin{bmatrix}
-2 & 0 & 0 \\
-1 & -3 & -1 \\
2 & 2 & 0
\end{bmatrix}
\]

(a) Is \( M \) symmetric?

(b) What are the eigenvalues of \( M \)? (Hint: don’t multiply the characteristic polynomial out; factor as much as you can as soon as you can.)

(c) For each eigenvalue above, give a basis for the eigenspace associated with that eigenvalue. Clearly indicate which basis goes with which eigenvalue.

(d) For each eigenvalue of \( M \), give the algebraic and geometric multiplicity of the eigenvalue and justify your answers.
(e) Is $M$ diagonalizable? If so, then write down the factorization of $M$ given by its similarity to a diagonal matrix and explicitly give $P$ and $D$ (you don’t have to compute $P^{-1}$). If not, tell why $M$ is not diagonalizable.

(f) Is $M$ invertible? Justify your answer by explicitly computing some quantity about $M$ and give a short explanation why the quantity justifies your answer. You do not need to compute $M^{-1}$ to answer this question.
10. Define the linear transformation $T: P_2 \to \mathbb{R}^3$ by $T(p) = \begin{bmatrix} p(-2) \\ p(0) \\ p(2) \end{bmatrix}$, where $p(a)$ is the number found by evaluating the polynomial $p(t)$ at $t = a$.

(a) Find the image of $p(t) = 2 + 3t$.

(b) Find the matrix $[T]_{B,E}$ for $T$ relative to the basis $B = \{1, 2t + 1, t^2 - t\}$ for $P_2$ and the standard basis for $\mathbb{R}^3$.

(c) Is $T$ bijective? Completely justify your answer.
11. The row reduced echelon form of 
\[
A = \begin{bmatrix}
-1 & 3 & -5 & 4 & 18 \\
1 & -2 & 4 & 0 & -7 \\
2 & 0 & 4 & -3 & -8 \\
5 & 1 & 9 & 2 & 2
\end{bmatrix}
\] 
is 
\[
\begin{bmatrix}
1 & 0 & 2 & 0 & -1 \\
0 & 1 & -1 & 0 & 3 \\
0 & 0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]. 

(a) Find a basis for the row space of \( A \).

(b) Find a basis for the column space of \( A \).

(c) Find the rank of \( A \). As always, justify your answer.