Exam 1 — Math 50 — Spring 2010

Write the best answer to each question in the box provided. For multiple choice and true/false questions, put your answer in the blank next to the question number.

No calculators. No notes.

1. _____ (4 pts) What is \( \lim_{x \to 3} x^2 - 2x + 1? \)
   (a) 0  (b) 1  (c) 2  (d) 3  (e) 4  (f) 5  (g) None of the above

2. _____ (4 pts) If \( \lim_{x \to 0^-} f(x) = 3 \), \( \lim_{x \to 0^+} f(x) = 3 \), and \( f(0) = 2 \), then what is \( \lim_{x \to 0} f(x)? \)
   (a) 0  (b) 2  (c) 3  (d) Does not exist.  (e) There is not enough information to give an answer.

3. _____ (4 pts) If \( \lim_{x \to c} f(x) = 0 \) and \( \lim_{x \to c} g(x) = 0 \), then \( \lim_{x \to c} \frac{f(x)}{g(x)} = \)
   (a) 0  (b) 1  (c) \( \infty \)  (d) \( \frac{0}{0} \)  (e) Does not exist.  (f) There is not enough information to give an answer.
4. Calculate the limits. If the limit does not exist, but is \(\infty\) or \(-\infty\), write “\(\infty\)” or “\(-\infty\)” as your answer, respectively; otherwise, write “DNE” for “does not exist”. (4pts each)

(a) \(\lim_{x \to 3} \frac{2x - 5}{x + 3}\)

(b) \(\lim_{x \to 0} x \csc x\)

(c) \(\lim_{x \to 1} \frac{x}{\sin \left( \frac{\pi}{2} x \right)}\)

(d) \(\lim_{x \to 4} \begin{cases} (x - 5)^3 & x \geq 4 \\ x + 7 & x < 4 \end{cases}\)

(e) \(\lim_{x \to 2^+} \sqrt{x^2 - 4}\)
(f) \[ \lim_{x \to 0^+} \frac{\ln(3x)}{x} \]

(g) \[ \lim_{x \to 0} \frac{\sin^2 3x}{2x^2} \]

(h) \[ \lim_{x \to 0} \frac{\sqrt{x + 2} - \sqrt{2}}{x} \]

(i) \[ \lim_{x \to 0} \frac{\tan(3x)}{x^2} \]

(j) \[ \lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} \]
5. (8 pts) Sketch a graph with the following characteristics. On the graph, label each discontinuity with its type (i.e., removable, jump, or infinite).

(a) $f(0)$ is undefined
(b) $\lim_{x \to 0^+} f(x) = 3$
(c) $\lim_{x \to 0^-} f(x) = 2$
(d) $\lim_{x \to 1} f(x) = \infty$
(e) $f$ has a removable discontinuity at $x = -1$
6. Let \( f(x) = x^2 \sin \left( \frac{1}{x} \right) \).

(a) (2pts) What is \( f(0) \)?

(b) (4pts) What is \( \lim_{x \to 0} f(x) \)? Explain your answer.

(c) (2pts) Is \( f(x) \) continuous at \( x = 0 \)?

7. (4pts) State the definition of what it means for a function \( f(x) \) to be continuous at a point \( x = c \).
8. Let \( f(x) = \frac{(x + 1)^2(2x)}{(x + 2)(x)(x - 1)} \).

(a) (12 pts) Calculate the limits. If the limit does not exist, but is \( \infty \) or \( -\infty \), write that as your answer; otherwise, write “DNE” for “does not exist”.

(1pt each)

i. \( \lim_{x \to -2^-} f(x) = \)

ii. \( \lim_{x \to -2^+} f(x) = \)

iii. \( \lim_{x \to 0^-} f(x) = \)

iv. \( \lim_{x \to 0^+} f(x) = \)

v. \( \lim_{x \to 1^-} f(x) = \)

vi. \( \lim_{x \to 1^+} f(x) = \)

(b) (6 pts) For what values of \( x \) is \( f(x) \) discontinuous? Label each discontinuity with its type (removable, jump, or infinite).

(c) (4 pts) Give a rough sketch of \( f(x) \). Label any vertical asymptotes.